



# NLO semi-inclusive Drell–Yan cross-section in quantum chromodynamics as a factorization analyzer

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## ABSTRACT

We evaluate in perturbative QCD the semi-inclusive Drell–Yan cross-section for the production of a single hadron accompanying the lepton pair. We demonstrate to one loop level a collinear factorization formula within the fracture functions approach. We propose such a process as a factorization analyzer in hadronic collisions. Phenomenological implications at the hadron colliders are briefly discussed.

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## 1. Introduction

The Drell–Yan [1] process has unique features among high energy hadronic reactions. The measurement of the relative cross-sections represents in fact a basic normalization in the LHC physics program, in particular by proving the universality of parton distribution functions as measured in deep inelastic scattering and by possibly pinning down non-standard evolution in the initial state parton cascade at small values of Bjorken variable  $x$ .

Universality of parton distribution functions follows directly from the factorization theorem for the Drell–Yan process. Despite some initial controversy on such a factorization in the early '80s, a series of papers [2] supported this hypothesis with specific model calculations and then a factorization proof was finally given in Refs. [3,4]. It was shown in particular that, when diagrams with soft gluon exchanges between active and spectators partons are summed over all possible cuts, soft gluon contributions decouple from the short distance cross-section, convalidating factorization at parton level as firstly conjectured by Drell and Yan. All the complications in proving the factorization can be related to the presence of two hadrons within the initial state.

The Drell–Yan process has furthermore two more appreciable features with respect to other short-distance hadron-induced cross-sections: (a) the perturbative scale can be accurately reconstructed by measuring the invariant mass  $Q^2$  of the lepton pair and (b) the final state is free from QCD-corrections thus providing a clean tool for the study of initial state radiation pattern.

Given these properties, it may become natural to investigate what happens if an additional hadron, accompanying the lepton pair, is identified in the final state:

$$P_1 + P_2 \rightarrow \gamma^* + h + X, \quad (1)$$

where  $P_1$  and  $P_2$  denote the incoming hadrons,  $h$  is the identified hadron in the final state,  $\gamma^*$  is the virtual (eventually electroweak) boson and  $X$  the unobserved part of the final state. The idea of using the Drell–Yan process as a *perturbative* trigger were first investigated in Ref. [5] and will be fully exploited also at the LHC [6,7]. If the invariant mass  $Q^2$  of the lepton pair is large enough so that perturbation theory applies, following the arguments of Ref. [8], the factorization property of the cross-section for the process in Eq. (1), should depend on the region of phase space in which the final hadron  $h$  is detected. In particular, if  $h$  is produced at sufficiently high transverse momentum,  $p_{h\perp}^2$ , then in such phase space regions (we refer to them as to *central*) standard perturbative QCD technique should be applicable, see for instance Ref. [9]. If  $h$  has instead a low  $p_{h\perp}^2$  and thus is detected in the so-called *target* fragmentation region, arguments against factorization have been already given in Refs. [10–12].

It is therefore highly desirable to have a standard perturbative framework which can be used as a “*factorization analyzer*” in both region of phase space. The first step is thus to provide a parton model formula which accounts for the production of an additional hadron. In the

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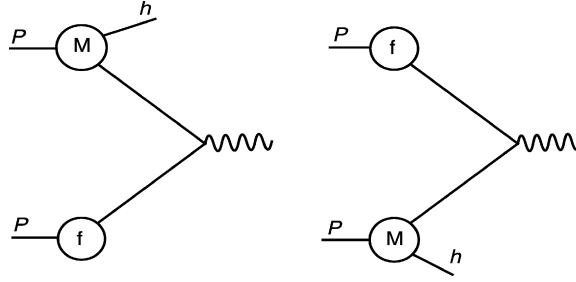


Fig. 1. A pictorial representation of the parton model formula, Eq. (4).

inclusive case the cross-section for the production of a lepton pair of mass  $Q^2$  in the collision of two hadrons, of momenta  $P_1$  and  $P_2$ , can be written as [1]

$$\frac{d\sigma^{\text{DY}}(\tau)}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \sum_q e_q^2 [f_q(x_1)f_{\bar{q}}(x_2) + (x_1 \leftrightarrow x_2)] \delta\left(1 - \frac{\tau}{x_1x_2}\right), \quad (2)$$

with  $S$  the hadronic center of mass energy,  $S = (P_1 + P_2)^2$ , and  $\tau = Q^2/S$  as in Ref. [13]. The sum runs over all quarks and antiquarks flavour but not on gluons which cannot directly couple to electroweak bosons. The parton distribution functions  $f_q(x)$  depend on the fractional momentum of the parton entering the hard scattering. In particular no scale dependence is indicated as appropriate in the naive parton model formula. In such an approach to the semi-inclusive Drell–Yan process of Eq. (1), where QCD higher order corrections are absent, we assume that the final state hadron is “non-perturbatively” produced in the target fragmentation region of  $P_1$  ( $\mathcal{R}_{T_1}$ ) or  $P_2$  ( $\mathcal{R}_{T_2}$ ) by means of a fracture function  $M_{h/P}^i(x, z)$ . These distributions give the conditional probability of finding a parton  $i$  with a fractional momentum  $x$  while an hadron  $h$ , with fractional momentum  $z$  of the incoming hadron momentum  $P$ , is detected in the target fragmentation region of  $P$ , see Ref. [14]. The collinear and soft factorization of these distributions in semi-inclusive DIS has been proven respectively in Refs. [12,15] and by an explicit  $\mathcal{O}(\alpha_s)$  QCD calculation in Ref. [16]. Supported by these results, continuous efforts have been devoted to the extraction diffractive parton distributions from HERA data. A combined analysis of both the diffractive and leading proton DIS data in terms of fracture functions has been presented in Ref. [17] while for more recent analysis we refer to Ref. [18]. Since no perturbative emissions are allowed in a parton model formula, we assume that “bare” fracture functions describe hadron production in the target fragmentation region  $\mathcal{R}_{T_1}$  of  $P_1$  if  $\theta_{\text{cm}} = 0$  and in  $\mathcal{R}_{T_2}$  of  $P_2$ , if  $\theta_{\text{cm}} = \pi$ , where  $\theta_{\text{cm}}$  is the relative angle between  $h$  and  $P_1$  in the hadronic center of mass frame. Phase space separation between target and central region is indeed unphysical and in this particular case also frame-dependent. However, this choice can be shown to be the more suitable in order to prove the factorization of the collinear singularities of the cross-sections. In the following we will consider the next-to-simple differential cross-sections for producing a lepton pair of invariant mass  $Q^2 \gg \Lambda_{\text{QCD}}^2$ , accompanied by an additional hadron  $h$  with fractional energy  $z = 2E_h/\sqrt{S}$  (defined in the hadronic center of mass frame) and integrated over its transverse momentum,  $p_{h\perp}^2$ . By defining the combination

$$M_q^h(x, z) = M_q^{h/P_1}(x, z) + M_q^{h/P_2}(x, z), \quad (3)$$

a straightforward generalization of Eq. (2) leads to the parton model formula for the semi-inclusive Drell–Yan process:

$$\frac{d\sigma^{\text{DY}}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2} \int_{\tau}^{1-\tau} \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \sum_q e_q^2 [M_q^h(x_1, z)f_{\bar{q}}(x_2) + (x_1 \leftrightarrow x_2)] \delta\left(1 - \frac{\tau}{x_1x_2}\right). \quad (4)$$

According to momentum conservation, the convolution integrals in Eq. (4) must satisfy the constraints  $1 - z \geq x_1x_2 \geq \tau$  and

$$x_1 + z \leq 1 \text{ in } \mathcal{R}_{T_1} \quad \text{and} \quad x_2 + z \leq 1 \text{ in } \mathcal{R}_{T_2}, \quad x_2 + z \leq 1 \text{ in } \mathcal{R}_{T_1} \quad \text{and} \quad x_1 + z \leq 1 \text{ in } \mathcal{R}_{T_2},$$

for the first and the second terms in square brackets of Eq. (4) respectively. The number of terms appearing in Eq. (4) is twice the number appearing in the inclusive case since each fracture functions selects its own fragmentation region. Eq. (4) represents for the moment only a factorization *conjecture* for the process we are considering and is sketched in Fig. 1. Our purpose is to show that Eq. (4) does survive to the inclusion of radiative corrections and that the factorization of collinear singularities is possible when the proper subtraction terms for bare fracture functions are added. The present QCD-based calculation deals with the standard soft gluon exchange between active partons but it is blind to soft gluon exchange between spectators whose effects are taken into account in Refs. [2–4] and could spoil the factorization, as suggested in Ref. [12]. In the presence of such factorization breaking effects, as emerged in recent phenomenological analysis, diffractive parton distribution extracted from HERA data do overestimate diffractive cross-sections measured at Tevatron by an order of magnitude [19]. Fracture functions appearing in Eq. (4) therefore cannot be related to the ones extracted from semi-inclusive DIS data. In a strong factorization breaking scenario the factorized form  $M \otimes f$  itself could in principle be questioned, since the exchanges of low momentum gluons cannot be uniquely absorbed neither in the definition of fracture nor of parton distribution functions so that they pertain to the reaction as a whole. The formalism we use cannot indeed penetrate inside the details of soft factorization, however it may constitute a quantitative next-to-leading order guideline for estimating the magnitude of its breaking when moving from expected non-factorizing phase space region at low  $p_{h\perp}^2$  to expected factorizing ones at high  $p_{h\perp}^2$ . The possible identification of an intermediate scale or range of scales at which this transition may occur would constitute an important matter of inspection insight in the dynamics of the factorization mechanism.

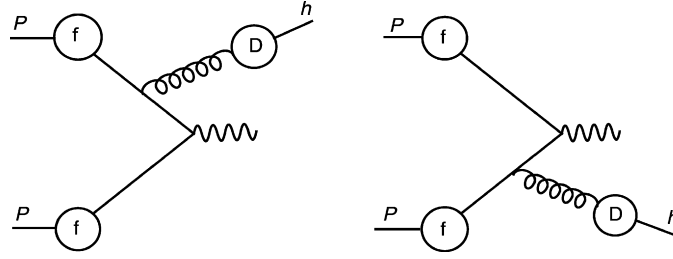


Fig. 2. Hadron production from initial state radiation, as described by Eq. (7).

## 2. Evaluation of NLO corrections

As outlined above, we have adopted the normalization and conventions as of Ref. [13]. The cross-sections for the leading order partonic sub-process  $q(p_1) + \bar{q}(p_2) \rightarrow \gamma^*$  is defined by:

$$\frac{d\hat{\sigma}_{q\bar{q}}^{(0)}}{dQ^2} = \delta(1-w) \frac{1-\epsilon}{2N_C}, \quad (5)$$

with  $N_C$  the number of colours and  $w = Q^2/s$  with  $s = (p_1 + p_2)^2$  the partonic center of mass energy. The space-time dimension is  $n = 4 - 2\epsilon$ . Eq. (5) is implicitly contained in parton model formula, Eq. (2) and Eq. (4). Moving to next order in perturbation theory requires the evaluation of the real emission diagrams  $q + \bar{q} \rightarrow \gamma^* + g$  and  $q + g \rightarrow \gamma^* + q$  along with virtual corrections to the Born amplitude. The results for the matrix elements squared, averaged over initial state colours and spins and summed over final ones, as well as the virtual contribution, can be found in Ref. [13]. Both the real and virtual contributions depend on the renormalization scale  $\mu_r^2$ . Moreover, the cross-section dependence on the variables associated to the produced hadron is entirely contained in fracture functions for both Born and virtual contributions.

So far no new ingredients have been added to a standard NLO perturbative calculations, the major subtleties coming from the two-particle final state of real emissions diagrams. In order not to obscure the renormalization procedure we restrict ourselves to the discussion of the  $q\bar{q}$  channel which indeed contains all the essential part of the calculations. In the following we will demonstrate that, at order  $\mathcal{O}(\alpha_s)$ , we can organize radiative corrections in such way that uncanceled (collinear) divergences can be reabsorbed into the “bare” distributions appearing in Eq. (4), in analogy with the semi-inclusive DIS calculation as performed in Ref. [16].

When the final state hadron is observed in  $\mathcal{R}_{T_1}$  or  $\mathcal{R}_{T_2}$  (for instance in hard single diffractive events in hadronic collisions), we assume, that it has been produced non-perturbatively from fracture functions. All the perturbative real radiation thus must be integrated over and virtual corrections added. The corrections to the semi-inclusive Drell-Yan process in the target fragmentation regions turn out to be the same as in the inclusive case. The two body phase space can be easily integrated in the partonic center of mass frame. It can be shown by explicit calculations that the double poles in the  $\epsilon$ -expanded results cancel in the sum of real and virtual term. This latter feature is peculiar of the  $q\bar{q}$  channel. For the singular contributions to the cross-section in the target regions we obtain

$$\frac{d\sigma_t^{\text{DY}}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2} \int_{\tau}^{1-z} \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \sum_q e_q^2 [M_q^h(x_1, z) f_{\bar{q}}(x_2) + (x_1 \leftrightarrow x_2)] \left[ \delta(1-w) - \frac{2}{\epsilon} \frac{\alpha_s(\mu_r^2)}{2\pi} P_{qq}(w) \left( \frac{4\pi\mu_r^2}{Q^2} \right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \right], \quad (6)$$

with  $w = \tau/x_1x_2$  and  $P_{ij}(w)$  the Altarelli-Parisi splitting function. Next we consider the production of the observed hadron  $h$  by the fragmentation of a, real, final state parton (i.e. a gluon) in the partonic sub-process. This  $\mathcal{O}(\alpha_s)$  production mechanism is sketched in Fig. 2. We address this phase space region as “central”, where the hadron  $h$  is allowed to be produced at high  $p_{h\perp}^2$ . In the collinear limit the “central” region collapses to the target fragmentation one. In general, such correction is expected of the form [20]

$$\frac{d\sigma_c^{\text{DY}}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2} \sum_q e_q^2 \int \frac{d\rho}{\rho} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} f_q(x_1) f_{\bar{q}}(x_2) D_g^h(z/\rho) \frac{d\hat{\sigma}_{q\bar{q}}^g}{dx_1 dx_2 d\rho}. \quad (7)$$

The variable  $\rho = 2E_k/\sqrt{s}$  is the partonic analogue of  $z$ , being  $E_k$  the energy of the outgoing gluon and  $d\hat{\sigma}_{q\bar{q}}^g$  the differential partonic cross-sections in the considered channel. The phase space integrations are performed in the hadronic rather than in the partonic center of mass frame. The final state gluon depends on its fractional energy  $\rho$  and on an angular variable  $y = (1 - \cos\theta_{\text{cm}})/2$ , being  $\theta_{\text{cm}}$  the relative angle between  $k$  and  $P_1$ . These variables are however not independent and constrained by the formula

$$\rho(y) = \frac{2(x_1x_2 - \tau)}{x_1 + x_2 + (2y - 1)(x_1 - x_2)}. \quad (8)$$

In order to keep compact the expressions it is useful to rewrite the  $\rho$  convolution as a  $y$  integral, with  $\rho$  depending on  $y$  via Eq. (8). The central contribution to the cross-sections therefore reads

$$\frac{d\sigma_c^{\text{DY}}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2} \sum_q e_q^2 \int_0^1 dy \int_{r_1(\tau, z; y)}^1 \frac{dx_1}{x_1} \int_{r_2(\tau, z; y)}^1 \frac{dx_2}{x_2} f_q(x_1) f_{\bar{q}}(x_2) D_g^h(z/\rho) \frac{d\hat{\sigma}_{q\bar{q}}^g}{dx_1 dx_2 dy} \frac{1}{\rho}. \quad (9)$$

The integration limits  $r_1$  and  $r_2$  are obtained by imposing momentum conservation and the condition  $\rho \geq z$  in order to guarantee that the energy of the parent parton is greater than the energy of the observed hadron:

$$r_1(\tau, z; y) = \frac{\tau + z(1-y)}{1-y}, \quad r_2(\tau, z; y) = \frac{\tau + x_1zy}{x_1 - z(1-y)}. \quad (10)$$

The emitted gluon in this region it is thus not allowed to be soft since it is required to produce the observed hadron. Once the invariants  $t$  and  $u$  appearing in the relevant matrix element squared are specified in this frame, we perform a standard  $\epsilon$ -expansion of the result in two disjoint singular limits, i.e. for  $y \rightarrow 0, 1$ . Retaining only  $\mathcal{O}(\epsilon^{-1})$  in the expansions we get the singular contributions to the cross-sections in this region of phase space:

$$\begin{aligned} \frac{d\sigma_c^{\text{DY}}(\tau)}{dQ^2 dz} &= \frac{4\pi\alpha^2}{9SQ^2} \int_{\tau+z}^1 \frac{dx_1}{x_1} \int_{\tau/(x_1-z)}^1 \frac{dx_2}{x_2} f_q(x_1) f_{\bar{q}}(x_2) D_g^h\left(\frac{zx_2}{x_1x_2-\tau}\right) \frac{\alpha_s(\mu_r^2)}{2\pi} \left(-\frac{1}{\epsilon}\right) \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_r^2}{Q^2}\right)^\epsilon \hat{P}_{(g)q \leftarrow q}\left(\frac{\tau}{x_1x_2}\right) \frac{x_2}{x_1x_2-\tau} \\ &+ \frac{4\pi\alpha^2}{9SQ^2} \int_{\tau/(1-z)}^1 \frac{dx_1}{x_1} \int_{z+\tau/x_1}^1 \frac{dx_2}{x_2} f_q(x_1) f_{\bar{q}}(x_2) D_g^h\left(\frac{zx_1}{x_1x_2-\tau}\right) \frac{\alpha_s(\mu_r^2)}{2\pi} \\ &\times \left(-\frac{1}{\epsilon}\right) \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_r^2}{Q^2}\right)^\epsilon \hat{P}_{(g)\bar{q} \leftarrow \bar{q}}\left(\frac{\tau}{x_1x_2}\right) \frac{x_1}{x_1x_2-\tau}. \end{aligned} \quad (11)$$

The first term in Eq. (11) is singular for  $y \rightarrow 0$  while the second for  $y \rightarrow 1$ . In the previous expression do appear unregularized Altarelli-Parisi splitting functions,  $\hat{P}_{(g)\bar{q} \leftarrow \bar{q}}(w)$ , since no interference with virtual contribution is present. Eqs. (6) and (11) thus represents all the singular collinear contributions to the semi-inclusive Drell-Yan cross-sections. If we were considering the inclusive Drell-Yan case, the only singularities would be the ones shown in Eq. (6). The subtraction of singular term in the partonic cross-sections would be performed by lumping the divergence into the bare parton distributions  $f$ . In the  $\overline{\text{MS}}$  subtraction scheme it reads:

$$f_i(\xi) = \int_{\xi}^1 \frac{du}{u} \left[ \delta_{ij} \delta(1-u) + \frac{1}{\epsilon} \frac{\alpha_s(\mu_r^2)}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_r^2}{\mu^2}\right)^\epsilon P_{ij}(u) \right] f_j\left(\frac{\xi}{u}, \mu^2\right). \quad (12)$$

In the previous equation  $\mu_r^2$  and  $\mu^2$  are respectively the renormalization and factorization scale. Since any observable built using Eq. (12) cannot depend on  $\mu^2$ , the derivative with respect to  $\ln \mu^2$  gives standard QCD evolution equations [21]. In the semi-inclusive case however there are additional singularities, Eq. (11), and the subtraction of Eq. (12) would not be sufficient to render the cross-sections infrared finite. Fracture functions, however, have been shown to have a more complex evolution equations with respect to the one of  $f$ . They contain an inhomogeneous term which accounts for hadron production in the target fragmentation region of the projectile by the showering of initial state radiation [14]. The analogous of Eq. (12) for bare fracture functions is obtained in the context of semi-inclusive deep inelastic scattering [16]:

$$\begin{aligned} M_i^h(\xi, \zeta) &= \int_{\xi/(1-\zeta)}^1 \frac{du}{u} \left[ \delta_{ij} \delta(1-u) + \frac{1}{\epsilon} \frac{\alpha_s(\mu_r^2)}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_r^2}{\mu^2}\right)^\epsilon P_{ij}(u) \right] M_j^h\left(\frac{\xi}{u}, \zeta, \mu^2\right) \\ &+ \int_{\xi}^{\xi/(\xi+\zeta)} \frac{du}{u} \frac{1}{1-u} \frac{u}{\xi} \frac{1}{\epsilon} \frac{\alpha_s(\mu_r^2)}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_r^2}{\mu^2}\right)^\epsilon \hat{P}_{(k)i \leftarrow j}(u) f_j\left(\frac{\xi}{u}\right) D_k^h\left(\frac{\zeta u}{\xi(1-u)}\right). \end{aligned} \quad (13)$$

The first term on r.h.s. of Eq. (13) has the same subtraction structure as for parton distribution, Eq. (12). The singularity is due do collinear radiation accompanying the active parton, while the hadron in the final state is non-perturbatively produced by the fracture functions itself. In the second term of Eq. (13) instead, the singularity is due to the observed hadron being collinear to the incoming hadron and generated by the perturbative fragmentation of the emitted collinear parton.

At this point we insert Eqs. (12) and (13) in the parton model result, Eq. (4). After some algebra and integral manipulation is easy but lengthly to show that Eq. (12) and the homogeneous term of Eq. (13) produce exactly the pole term in Eq. (6) with the opposite sign. The inhomogeneous term in Eq. (13) instead reproduce the singular term in Eq. (11). All the singularities therefore cancel and we are left with

$$\begin{aligned} \frac{d\sigma^{\text{DY}}(\tau)}{dQ^2 dz} &= \frac{4\pi\alpha^2}{9SQ^2} \sum_q e_q^2 \int_{\tau}^{1-z} \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \cdot [M_q^h(x_1, z, \mu_F^2) f_{\bar{q}}(x_2, \mu_F^2) + (x_1 \leftrightarrow x_2)] \left[ \delta\left(1 - \frac{\tau}{x_1x_2}\right) + \frac{\alpha_s(Q^2)}{2\pi} C_{qq}\left(\frac{\tau}{x_1x_2}, \frac{\mu_F^2}{Q^2}\right) \right] \\ &+ \frac{4\pi\alpha^2}{9SQ^2} \sum_q e_q^2 \int_0^1 dy \int_{r_1}^1 \frac{dx_1}{x_1} \int_{r_2}^1 \frac{dx_2}{x_2} [f_q(x_1, \mu_F^2) f_{\bar{q}}(x_2, \mu_F^2) + (x_1 \leftrightarrow x_2)] D_g^h\left(\frac{z}{\rho}, Q^2\right) \frac{\alpha_s(Q^2)}{2\pi} K_{q\bar{q}}^g\left(z, y, \frac{\tau}{x_1x_2}, \frac{\mu_F^2}{Q^2}\right). \end{aligned} \quad (14)$$

The function  $C_{q\bar{q}}$  and  $K_{q\bar{q}}^g$  are infrared-finite and depend explicitly on the factorization scale  $\mu_F^2$ . All bare distributions are replaced by renormalized as indicated by the explicit factorization scale dependence. In particular,  $C_{q\bar{q}}$  is the same as in the inclusive Drell-Yan case whereas  $K_{q\bar{q}}^g$  is specific of the semi-inclusive process. Its explicit form along with the coefficient for the gluon initiated channel will be reported in a separate paper, as well as a more detailed description of the calculation. In Eq. (14) we may set  $\mu_F^2 = Q^2$  in order to remove potentially large logarithmic corrections of the type  $\ln(\mu_F^2/Q^2)$  from the coefficient functions and resum them by using the appropriate evolution equations for  $f$  and  $M$ .

### 3. Conclusions

We have shown, with a fixed  $\mathcal{O}(\alpha_s)$  perturbative QCD calculation, that the partonic cross-sections for the semi-inclusive Drell–Yan process do factorize at the collinear level. We therefore implicitly confirm the general widespread idea indicating that soft exchanges between active and spectators partons [10] as responsible for factorization breaking in semi-inclusive reactions.

We would like to conclude by listing a few remarks and proposals. As opposed to full inclusive observables, semi-inclusive ones in hadronic collisions are affected by soft gluon exchange contributions and could therefore act as factorization analyzers at phenomenological level. In the central region, high  $p_{h\perp}^2$  hadron production should follow the pattern predicted by perturbative QCD. When the detected hadron is, instead, a low  $p_{h\perp}^2$  proton, diffractive processes may occur [22] and the partonic structure of the color-singlet exchanged object may be studied [23]. In these particular cases a non-universality of diffractive parton distributions, as taken from diffractive DIS and hadronic collisions, was suggested in Ref. [11]. This was experimentally reported in Ref. [19]. It would be interesting to establish with a comparison with the data whether a factorization breaking shows up only in a diffractive kinematic regime or if it manifests itself also in processes with a gapless final state containing a single hadron as well in the target fragmentation region. It would be also interesting to test factorization in light mesons production which is sensitive to the soft, high multiplicity, fragmentation process.

The present work is well suited to analyze single diffractive  $W^\pm/Z$  production and can be easily generalized to double hadron production. The evaluation of a double hadron production cross-section needs a full  $\mathcal{O}(\alpha_s^2)$  QCD calculation. However, an approximate result could be obtained if one considers two hadron at low  $p_{h\perp}^2$  observed in opposite fragmentation regions with respect to the incoming hadrons. As it happens at NLO, higher order corrections for this process should be the same as for inclusive Drell–Yan process, once the proper kinematic is taken into account. In Ref. [24] we have suggested an analogous formula, in leading logarithmic approximation, for double-inclusive Drell–Yan production which also includes the additional dependence on the invariant momentum transfer at the proton's vertex  $t_1$  and  $t_2$ .

Finally we are thinking to a generalization of the present approach to include gluon initiated hard processes [25] whose relevance in diffractive Higgs production was first suggested in Ref. [26].

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